

Lesson

35

Volume of Rectangular Prisms and Cubes

Measures of length can be used to calculate area—the space inside a two-dimensional figure. Area is the square units that cover the space.

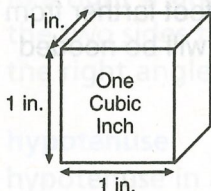
Measures of length are also used to calculate volume. **Volume** is a measure of the amount of space inside a three-dimensional object. Volume is measured in three-dimensional units or **cubic units**—cubic inches (cu in.), cubic feet (cu ft), cubic yards (cu yd), and cubic centimeters (cu cm), and other metric units. Each of these units is a cube with identical square sides.

volume

the measure of the amount of space inside a three-dimensional figure

cubic units

used to measure the volume of a three-dimensional figure; units needed to fill the space inside a three-dimensional figure

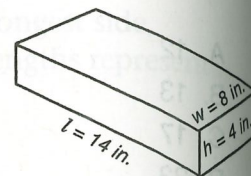


rectangular prism

a three-dimensional figure in which all sides are rectangles and all corners are square

One of the most common three-dimensional objects is the **rectangular prism**. Boxes, crates, and rooms are examples of rectangular prisms. To find the volume of these objects, multiply the length times the width times the height.

Example 1 What is the volume of a rectangular container that is 14 inches long, 8 inches wide, and 4 inches high?



Multiply length times width times height.

$$14 \text{ in.} \times 8 \text{ in.} \times 4 \text{ in.} = 448 \text{ cubic inches}$$

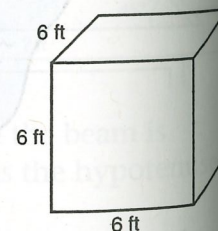
Multiply the numbers as shown.

$$14 \times 8 \times 4 \text{ ENTER } 448.$$

The volume of the container is **448 cubic inches**. In other words, you could fit 448 one-inch cubes inside the container.

A **cube** is a special rectangular prism, so you can find the volume in the same way. However, since each side has the same measure, you are multiplying the same number three times, or “cubing” it.

Example 2 A shipping crate has the shape of a cube that is 6 feet long on each edge. What is its volume in cubic feet?



$$\begin{aligned} \text{Volume} &= \text{length} \times \text{width} \times \text{height} \\ &= 6 \text{ ft} \times 6 \text{ ft} \times 6 \text{ ft} \\ &= 216 \text{ cubic feet} \end{aligned}$$

cube

a rectangular solid with six square sides (faces; all edges of equal length)



Using a scientific calculator, you can cube a number using the \wedge key. Enter the number you want to multiply, press \wedge , enter the exponent 3, and press the ENTER key.

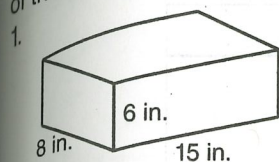
$$6 \wedge 3 \text{ ENTER } 216.$$

The volume of the shipping crate is **216 cubic feet**.

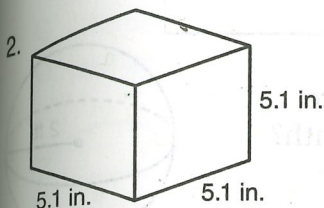
GED® Practice

Directions: Choose or write the answer to each question.

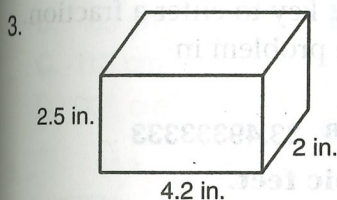
For problems 1 through 3, find the volume of the prism.



- A. 48 cu in. C. 120 cu in.
B. 90 cu in. D. 720 cu in.



- A. 15.3 cu in. C. 132.651 cu in.
B. 26.01 cu in. D. 156.06 cu in.



- A. 8.7 cu in. C. 21 cu in.
B. 10.5 cu in. D. 27.89 cu in.

4. What is the volume in cubic inches of a rectangular solid with these measurements: length = 15 inches, width = 24 inches, and height = 4 inches?

5. What is the volume in cubic yards of a cube that measures 3 yards on each side?

Question 6 refers to the following figure.



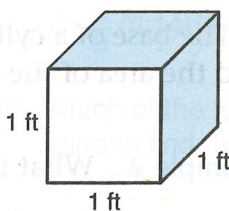
A contractor has hired a trash hauling service to remove trash from a job site. The trash bins are rectangular containers with the measurements shown in the diagram. How many cubic feet of trash can each container hold?

- A. 192 cu ft C. 568 cu ft
B. 288 cu ft D. 576 cu ft

7. What is the volume in cubic yards of a rectangular container with these measurements: length = 6 feet, width = 3 feet, and height = 9 feet? (Hint: 1 cu yd = 27 cu ft.)

- A. 6 cu yd C. 54 cu yd
B. 18 cu yd D. 162 cu yd

Question 8 refers to the following figure.



8. How many cubic inches are in 1 cubic foot? (Hint: 1 ft = 12 in.)

- A. 36 cu in. C. 864 cu in.
B. 144 cu in. D. 1,728 cu in.

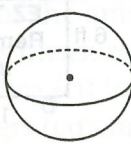

Answers start on page 794.

Lesson

36

Volume of Cylinders and Spheres

Volume is a measure in cubic units of the amount of space inside a three-dimensional (or "solid") object. Two common solid shapes are a **sphere** and a **cylinder**.

Sphere	Cylinder
	
$V = \frac{4}{3} \times \pi \times \text{radius}^3$ $= \frac{4}{3} \pi r^3$	$V = (\pi \times \text{radius}^2) \times \text{height}$ $= \pi r^2 h$

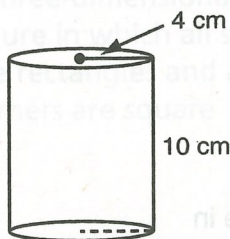


Tip

Save time by memorizing the basic formulas for perimeter, area, and volume.

sphere

a solid (three-dimensional) figure formed by all points in space that are the same distance (radius) from a given point (center)

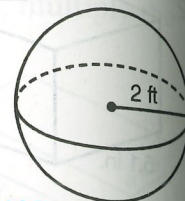


cylinder

a solid (three-dimensional) figure with two congruent circular bases and a curved surface



Example 1 What is the volume of the sphere shown at right to the nearest tenth?



Step 1 Choose the formula.

$$V = \frac{4}{3} \pi r^3$$

Step 2 Substitute and solve.

$$= \frac{4}{3} (3.14) (2)^3 = 33.493$$

33.493 to the nearest tenth is 33.5 cubic feet.

Use the \wedge key to cube a number and the $\frac{n}{d}$ key to enter a fraction. Use the keystrokes below to solve the sphere problem in Example 1.

$$4 \frac{n}{d} 3 \times 3 \cdot 14 \times 2 \wedge 3 \text{ ENTER } 33.49333333$$

The volume of the sphere is about **33.5 cubic feet**.

The base of a cylinder is a circle. To find the volume of a cylinder, find the area of the circle (πr^2) and multiply this area by the height (h).

Example 2 What is the volume of the cylinder shown at left?

Step 1 Choose the formula.

$$V = (\text{area of a circle}) \times h = \pi r^2 h$$

Step 2 Substitute.

$$V = 3.14 \times 4^2 \times 10$$

Step 3 Solve.

$$V = 3.14 \times 16 \times 10 = 502.4 \text{ cu cm}$$

You can use either the x^2 or \wedge key when solving volume problems on a calculator. Try the keystrokes below to solve the cylinder problem in Example 2.

$$3 \cdot 14 \times 4 x^2 10 \text{ ENTER } 502.4 \quad \text{or}$$

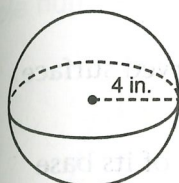
$$3 \cdot 14 \times 4 \wedge 2 \times 10 \text{ ENTER } 502.4$$

The volume of the cylinder is about **502.4 cubic centimeters**.

Directions: Choose or write the answer to each question. You may use your calculator.

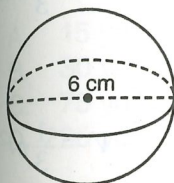
For problems 1 through 5, find the volume of the figure to the nearest whole number.

1.



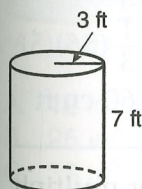
- A. 17 in³
- B. 67 in³
- C. 268 in³
- D. 804 in³

2.



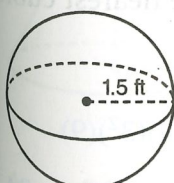
- A. 38 cm³
- B. 85 cm³
- C. 113 cm³
- D. 905 cm³

3.



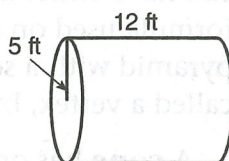
- A. 86 ft³
- B. 198 ft³
- C. 462 ft³
- D. 798 ft³

4.



- A. 9 ft³
- B. 14 ft³
- C. 36 ft³
- D. 113 ft³

5.



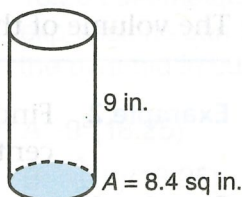
- A. 188 ft³
- B. 236 ft³
- C. 377 ft³
- D. 942 ft³

6. What is the volume of a sphere with a diameter of 9 feet? Use 3.14 for π . Round the answer to the nearest whole number.

7. Which of the following expressions can be used to find the volume in cubic inches of a ball with a diameter of 7 inches?

- A. $\frac{4}{3}(3.14)(3.5)^2$
- B. $\frac{4}{3}(3.14)(7)^2$
- C. $\frac{4}{3}(3.14)(7)^3$
- D. $\frac{4}{3}(3.14)(3.5)^3$

8. Which of the following expressions can be used to find the volume of the cylinder in cubic inches?



- A. $(8.4)9$
- B. $(8.4^2)(9)$
- C. $(3.14)(8.4)(9)$
- D. $(3.14)(8.4^2)(9)$

Answers start on page 795.

1 Review the Skill

MATH CONTENT TOPICS: Q.2.a, Q.2.e, Q.5.a, Q.5.b, Q.5.c, A.2.a, A.2.b, A.2.c
MATH PRACTICES: MP.1.a, MP.1.b, MP.1.d, MP.1.e, MP.2.a, MP.2.c, MP.3.a, MP.3.b, MP.4.a, MP.4.b, MP.5.b

A **solid figure** is a 3-dimensional figure. The **volume** of a solid figure is the amount of space it occupies. The volume of a prism or cylinder is the product of the area of its base and its height. Volume is measured and shown in cubic units (³).

The **surface area** of a solid figure is the sum of the areas of its two bases *and* the area of its lateral surfaces. The surfaces of a prism are polygons. Use formulas for the areas of triangles, rectangles, and other polygons to compute the surface area of a prism. Such formulas will help you to determine the complete dimensions of a figure. For example, if you know the area of the base of a cylinder and its surface area, you can calculate the height.

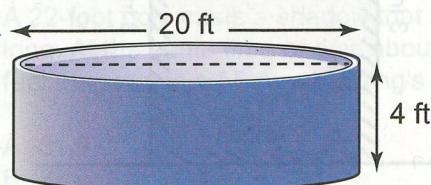
2 Refine the Skill

By refining the skills of computing the surface area and volume of prisms and cylinders, you will improve your study and test-taking abilities, especially as they relate to the GED® Mathematical Reasoning Test. Study the information below. Then answer the questions that follow.

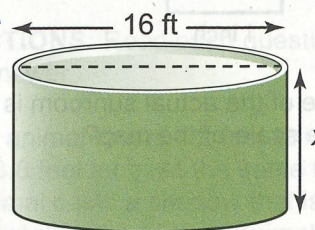
a The volume of the two pools is the same. For question 1, since the diameter and height are both given for Pool A, find the volume of this pool.

b Remember that the formula for volume of a cylinder uses the radius, not the diameter. For both questions, divide the diameter by 2 to find the radius.

Genevieve wants to buy an above-ground swimming pool. She is trying to decide between two models, shown below. Each model holds the same amount of water. The height of Pool B is represented by x .



Pool A



Pool B

- What is the volume, to the nearest cubic foot, of each pool?
 - 400
 - 1,005
 - 1,256
 - 5,024
- What is the height, in feet, of Pool B?
 - 4.25 ft
 - 6.25 ft
 - 8.0 ft
 - 10.0 ft

TEST-TAKING TIPS

Questions relating to solid figures and volume may ask you to solve for diameter, radius, or area of a base, or the length, width, or height of a figure. Read each problem carefully to decide what to solve.

3 Master the Skill

DIRECTIONS: Read each question, and choose the **best** answer.

3. A grocery store sells Italian sausage in the cylindrical tube-shaped package shown below.



10 in.

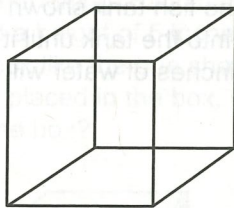
If the radius of the package is about 1.5 inches, how many cubic inches is the package?

- A. 70.65
B. 93.40
C. 104.67
D. 141.30
4. A cylinder has a diameter of 10 meters and a height of 7 meters. What is the volume of the cylinder?

- A. 109.9 m^3
B. 549.5 m^3
C. 1725.4 m^3
D. 2198 m^3

DIRECTIONS: Study the information and figure, read each question, and choose the **best** answer.

The cube-shaped box below has a length, width, and height of 18 inches.



5. What is the volume of the box to the nearest cubic foot?

- A. 3
B. 36
C. 324
D. 5,832

6. What is the surface area of the box to the nearest square foot?

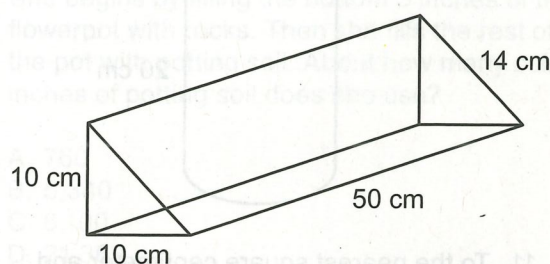
- A. 14
B. 19
C. 324
D. 1944

DIRECTIONS: Read each question, and choose the **best** answer.

7. Anaya's perfume bottle is a rectangular prism. The bottle is 2.5 centimeters wide and 13 centimeters tall. If the volume of the bottle is 97.5 cubic centimeters, what is the length of the perfume bottle to the nearest whole centimeter?

- A. 2.5 cm
B. 3.0 cm
C. 3.3 cm
D. 7.5 cm

8. A shipping package has the shape of the triangular prism shown below.



Assuming the package has no gaps or overlaps, what area of cardboard is needed to produce the shipping package?

- A. $1,300 \text{ cm}^2$
B. $1,750 \text{ cm}^2$
C. $1,800 \text{ cm}^2$
D. $2,500 \text{ cm}^2$

9. Calvin says that the surface area of a cube is always greater than its volume. Which side length of a cube shows that Calvin is incorrect?

- A. 0.5 ft
B. 2 ft
C. 5 ft
D. 8 ft

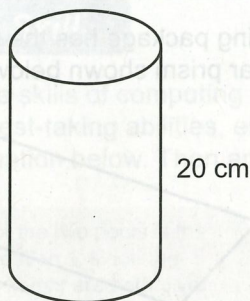
10. What is the diameter of a cylinder with a volume of 235.5 cubic inches and a height of 3 inches?

- A. 2.5 in.
B. 5 in.
C. 10 in.
D. 25 in.

★ Spotlighted Item: **DROP-DOWN**

DIRECTIONS: Study the information and figure, read each question, and choose the **best** answer from the drop-down list.

An open-top flour canister has a circumference of 31.4 centimeters and a height of 20 centimeters. The bottom and outside are made of cardboard and the lid is made of plastic.



11. To the nearest square centimeter and assuming there is no overlap, the area of cardboard needed to make the canister is **Drop-down** cm^2 .

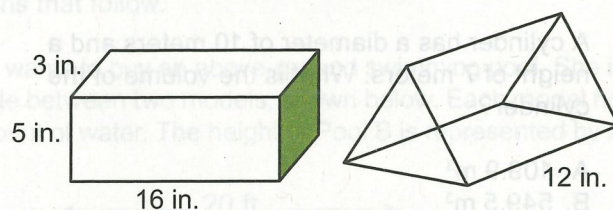
A. 157 B. 314 C. 628 D. 707

12. A second canister holds the same volume of flour as the canister to the left but is 15 centimeters high. To the nearest tenth of a centimeter, the circumference of the second canister is **Drop-down** cm.

A. 33.3 B. 36.2 C. 72.5 D. 133.3

DIRECTIONS: Study the information and figures, read the question, and choose the **best** answer from the drop-down list.

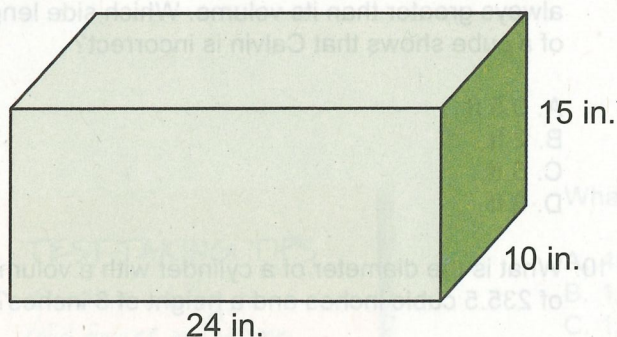
The two prisms have the same volume.



13. The area of the base of the triangular prism is **Drop-down** in.^2 .

A. 10 B. 11 C. 15 D. 20

DIRECTIONS: Study the figure, read the question, and choose the **best** answer.



14. Alyssa has the fish tank shown to the left. If she pours water into the tank until it is half full, how many cubic inches of water will be in the tank?

A. 7,200
B. 3,600
C. 1,800
D. 450

DIRECTIONS: Study the information, read the question, and choose the **best** answer.

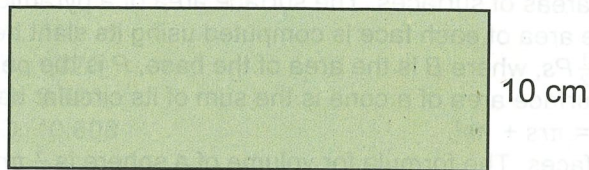
A can of soda is 3 inches in diameter and holds 28.26 cubic inches

15. What is the height of the can?

A. 1
B. 3
C. 4
D. 6

DIRECTIONS: Study the information and figure, read each question, and choose the **best** answer.

A company makes paper cups by rolling rectangular pieces of paper stock, like the one shown below, into cylinders. Each cup has a height of 10 cm.



16. If a cup has a radius of 3.5 cm, what is the volume of the cup in cubic centimeters?

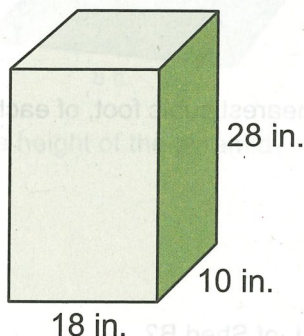
A. 384.7
B. 219.8
C. 192.3
D. 109.9

17. If a cup has a radius of 3.5 cm, what is the lateral area of the cup in square centimeters?

A. 35
B. 70
C. 110
D. 220

DIRECTIONS: Read the question, and choose the **best** answer.

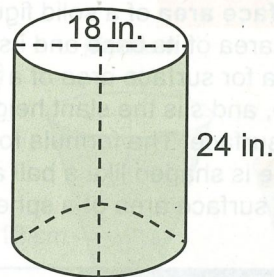
18. A food service company ships boxes that contain 40 cans of shortening. Each can has a radius of 2 inches and a height of 5 inches. The cans are placed in a box like the one shown below. After the cans are placed in the box, how much empty space is in the box?



A. 2,512 cubic inches
B. 2,528 cubic inches
C. 3,784 cubic inches
D. 4,973 cubic inches

DIRECTIONS: Read the question, and choose the **best** answer.

19. Kaya is planting flowers in the cylindrical-shaped flowerpot shown below.

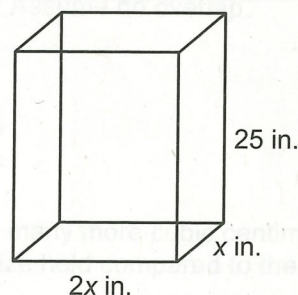


She begins by filling the bottom 3 inches of the flowerpot with rocks. Then she fills the rest of the pot with potting soil. About how many cubic inches of potting soil does she use?

A. 760
B. 5,340
C. 6,100
D. 21,360

DIRECTIONS: Study the information and figure, read each question, and choose the **best** answer.

A rectangular prism with volume 4,050 cubic inches has a base that is twice as long as it is wide. Its height is 25 inches.



20. What is the length of the prism?

A. 4.5 in.
B. 9 in.
C. 18 in.
D. 27 in.

21. What is the surface area of the prism?

A. 1,674 in.²
B. 1,198 in.²
C. 837 in.²
D. 599 in.²