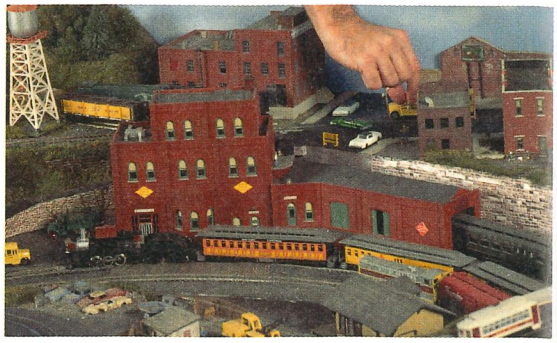
Scale Models



A model that is a careful, reduced copy of an actual object is called a **scale model.** You have probably seen scale models of cars, trains, and airplanes. The size-change factor in scale models is usually called the **scale factor.**

Dollhouses often have a scale factor of $\frac{1}{12}$. You can write this as "$\frac{1}{12}$ of actual size," "scale 1 : 12," "$\frac{1}{12}$ scale," or as a proportion:

$$\frac{\text{dollhouse length}}{\text{real house length}} = \frac{1 \text{ inch}}{12 \text{ inches}}$$

All the dimensions of an E-scale model railroad are $\frac{1}{96}$ of the actual size. The scale factor is $\frac{1}{96}$. We can write this as "scale 1 : 96," or as a proportion:

$$\frac{\text{model railroad length}}{\text{real railroad length}} = \frac{1 \text{ inch}}{96 \text{ inches}}$$

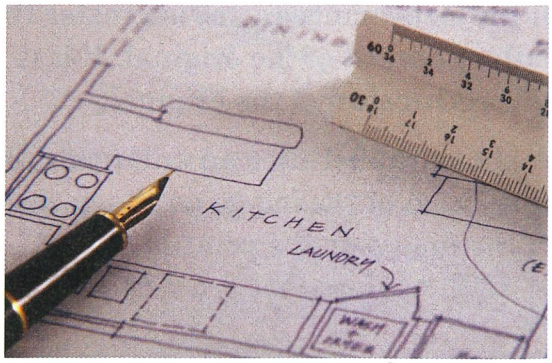
We can also write this as "scale: $\frac{1}{8}$ inch represents 1 foot," or as "scale: 0.125 inch represents 12 inches." To see this, write

$$\frac{\frac{1}{8}}{12} = \frac{\frac{1}{8} * 8}{12 * 8} = \frac{1}{96}$$

$\frac{1}{8}$ inch : 12 inches is the same as 1 inch : 96 inches.

Scale Drawings

The size-change factor for scale drawings is usually called the **scale.** If an architect's scale drawing shows "scale $\frac{1}{4}$ inch : 1 foot" or "scale: $\frac{1}{4}$ inch represents 1 foot," then $\frac{1}{4}$ inch on the drawing represents 1 foot of actual length.



$$\frac{\text{drawing length}}{\text{real length}} = \frac{\frac{1}{4} \text{ inch}}{1 \text{ foot}}$$

Since 1 foot = 12 inches, we can rename

$$\frac{\frac{1}{4} \text{ inch}}{1 \text{ foot}} \text{ as } \frac{\frac{1}{4} \text{ inch}}{12 \text{ inches}}.$$

Multiply by 4 to change this to an easier fraction:

$$\frac{\frac{1}{4} \text{ inch} * 4}{12 \text{ inches} * 4} = \frac{1 \text{ inch}}{48 \text{ inches}}.$$

The drawing is $\frac{1}{48}$ of the actual size.

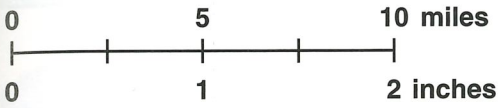
Map Scales

Cartographers (mapmakers) show large areas of land and water in small areas on paper. The size-change factor for a map is usually called the **map scale.** Using a map and a map scale, you can estimate actual distances. Different maps use different scales.

If a map scale is 1 : 24,000, then every length on the map is $\frac{1}{24{,}000}$ of the actual length, and any real distance is 24,000 times the distance shown on the map.

$$\frac{\text{map distance}}{\text{real distance}} = \frac{1}{24{,}000}$$

On the map scale below, the length of the bar stands for 10 actual miles. Half the length of the bar stands for 5 actual miles.



Since the bar is 2 inches long and stands for 10 actual miles, the map scale can also be written:

$$\frac{\text{map distance}}{\text{real distance}} = \frac{2 \text{ inches}}{10 \text{ miles}}.$$

You can find the exact size-change factor for a map with this scale by converting miles to inches in the proportion.

1 mile = 5,280 feet and 1 foot = 12 inches.
So, 10 miles = 52,800 feet = 52,800 * 12 inches = 633,600 inches.

$$\frac{\text{map distance}}{\text{real distance}} = \frac{2 \text{ inches}}{633{,}600 \text{ inches}} = \frac{1 \text{ inch}}{316{,}800 \text{ inches}}$$

The size-change factor is 1: 316,800, or $\frac{1}{316{,}800}$.

Caution: You may see scales written with an equal sign, such as "$\frac{1}{4}$ inch = 1 foot." But $\frac{1}{4}$ inch is certainly not equal to 1 foot, so "$\frac{1}{4}$ inch = 1 foot" is not mathematically correct. What is meant is that $\frac{1}{4}$ inch on the map or scale drawing stands for 1 foot in the real world.

Did You Know?

The U.S. Geological Survey (USGS) has made a detailed set of maps that covers the entire area of the U.S.

Their best known maps have a scale of 1:24,000 (1 inch represents 24,000 inches = 2,000 feet). Each of these maps covers an area of between 49 and 65 square miles.

Check Your Understanding

1. The side of a square is 2.5 cm. A copier is used to make an enlargement of the square. The size-change factor is 3.

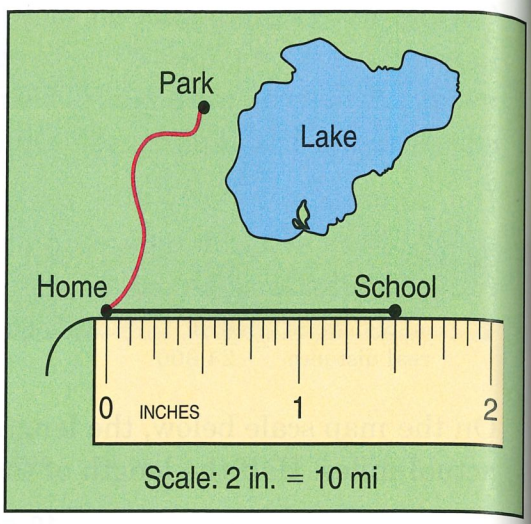
a. What is the side of the enlarged square?

b. What is the perimeter of the enlarged square?

Check your answers on page 418.

Finding Distances Using a Map Scale

There are many ways to find actual distances using a map. This method requires a ruler and string.



Step 1: Measure the map distance.

If the distance is along a straight path, such as the distance from Home to School, use the ruler to measure the map distance directly.

If the distance is along a curved path, such as the distance from Home to the Park:

- Lay the string along the path. Mark the beginning and ending points on the string.
- Straighten out the string. Use a ruler to measure between the beginning and ending points.

Step 2: Use the map scale to find the real distance for the map distance you measured. For example, the map distance from Home to School is 1.5 inches and the scale is 10 miles to 2 inches. To find the real distance we can solve this number model:

$$\frac{\text{real distance}}{1.5 \text{ inches}} = \frac{10 \text{ miles}}{2 \text{ inches}}$$

One way to solve this number model is to change the ratio to an equivalent n-to-1 ratio.

$$\frac{\text{real distance}}{1.5 \text{ inches}} = \frac{10 \text{ miles}}{2 \text{ inches}} = \frac{10 \text{ miles} \div 2}{2 \text{ inches} \div 2} = \frac{5 \text{ miles}}{1 \text{ inch}}$$

1 inch on the map stands for 5 miles, so 1.5 inches must stand for $1.5 * 5$ miles, or 7.5 miles.

Check Your Understanding

Look at the map at the right.
The map scale is $\frac{\text{map distance}}{\text{real distance}} = \frac{1 \text{ inch}}{200 \text{ miles}}$

1. What is the actual distance between
   - **a.** New Orleans and Charlotte?
   - **b.** Mobile and Birmingham?
   - **c.** Charlotte and Tampa?
   - **d.** Tampa and Memphis?

Check your answers on page 418.