Lesson

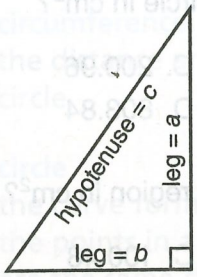
34

The Pythagorean Theorem

The Pythagorean Theorem explains the special relationship between the **legs** (the two shorter sides) and the **hypotenuse** (the longest side) of a right triangle. It states that in a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.

$a^2 + b^2 = c^2$, where a and b are the legs and c is the hypotenuse of a right triangle

You can use the Pythagorean Theorem to solve various problems involving right triangles.



leg
in a right triangle, one of the two sides that form the right angle

hypotenuse
hypotenuse in a right triangle, the side opposite the right angle; the longest side in a right triangle

Example 1 The lengths of the sides of a triangle are 5, 6, and 9 inches. Is the triangle a right triangle?

Step 1 Identify the legs and the hypotenuse.

In order for the triangle to be a right triangle, its longest side, 9 inches, must be the hypotenuse. Let the other lengths represent the legs, $a = 5$ and $b = 6$.

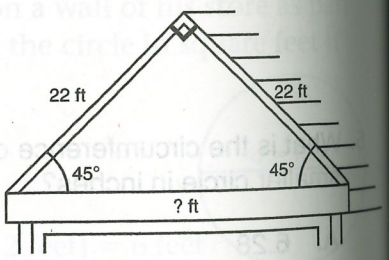
Step 2 Use the Pythagorean Theorem.

If the three sides make the Pythagorean Theorem true, then this is a right triangle.

$$a^2 + b^2 = c^2$$
$$5^2 + 6^2 \stackrel{?}{=} 9^2$$
$$25 + 36 \stackrel{?}{=} 81$$
$$61 \neq 81$$

The triangle is **not a right triangle**.

Example 2 A beam across the front of a garage needs to be replaced. Using the diagram, what is the length of the beam to the nearest foot?



Step 1 Identify the legs and the hypotenuse. Since the beam is opposite the right angle, you know that it is the hypotenuse.

Step 2 Use the Pythagorean Theorem.

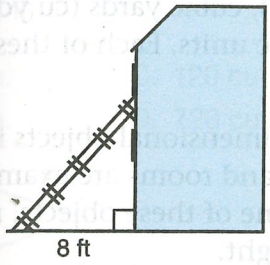
$$a^2 + b^2 = c^2$$
$$22^2 + 22^2 = c^2$$
$$484 + 484 = c^2$$
$$968 = c^2$$
$$\sqrt{968} = c \approx 31$$

The beam is about **31 feet** in length.

GED® Practice

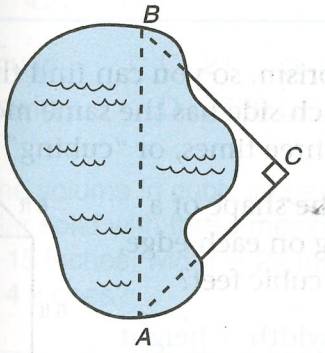
Directions: Choose or write the answer to each question.

1. The bottom of a ladder is placed 8 feet from the wall of a house. The wall and the ground form a right angle. If the ladder is 15 feet in length, how far up the wall does it reach (to the nearest foot)?



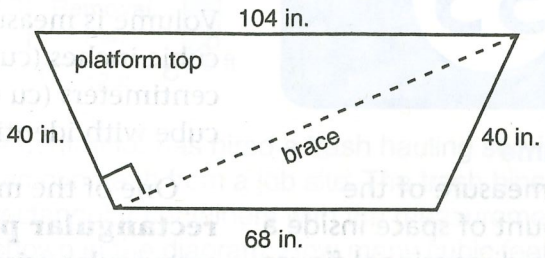
A. 12
B. 13
C. 17
D. 23

2. Bill wants to find the width of the pond. He places stakes at A and B and then finds C so that C is a right angle. This makes $\triangle ABC$ a right triangle. If $AC = 60$ feet and $BC = 80$ feet, about how far is it from A to B?



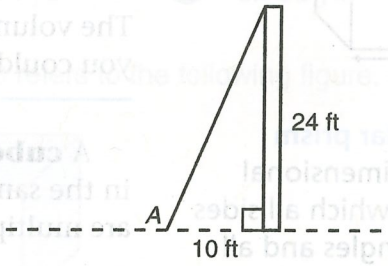
A. from 85 to 95 ft
B. from 95 to 105 ft
C. from 105 to 115 ft
D. more than 115 ft

3. Jan builds a platform with a top in the shape of a trapezoid. The plans call for a diagonal brace to be added. What is the length of the brace to the nearest inch?



A. 64
B. 88
C. 96
D. Not enough information is given.

4. The pole is perpendicular to the ground. A wire is stretched from the ground to the top of the pole. If point A is moved 8 feet farther from the pole, how much more wire will be needed to reach the top of the pole?



A. 4 ft
B. 6 ft
C. 26 ft
D. 30 ft

5. The lengths of the sides of a triangle are 1, $1\frac{1}{3}$, and $1\frac{2}{3}$. Is the triangle a right triangle? Circle the answer that completes the sentence.

The triangle | is | is not | a right triangle.

Answers start on page 794.

LESSON 2

Pythagorean Theorem

Use with *Student Book* pp. 96–97

MATH CONTENT TOPICS: Q.4.a, A.4.a, Q.4.e, A.4.b
MATH PRACTICES: MP.1.a, MP.1.b, M.P.2.b, MP.3.c, MP.4.b, MP.5.a

1 Review the Skill

As you know, a **right triangle** has a right angle. The legs (shorter sides) and **hypotenuse** (longer side) of a right triangle have a special relationship that can be described by the **Pythagorean Theorem**. It states that, in any right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse. It is expressed in equation form as $a^2 + b^2 = c^2$. You can use this theorem to find a missing length of a right triangle.

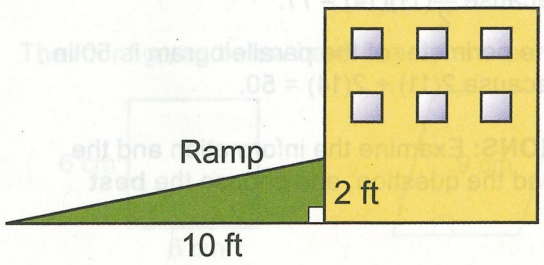
2 Refine the Skill

By refining the skill of using the Pythagorean Theorem to solve for the missing side of a right triangle, you will improve your study and test-taking abilities, especially as they relate to the GED® Mathematical Reasoning Test. Examine the diagram and strategies below. Then answer the questions that follow.

a The measurements of the legs of the right triangle are given. Solve for the hypotenuse to find the length of the ramp.

b To solve question 2, substitute 12 for 10 and solve for the hypotenuse. Remember that the hypotenuse is always the longest side of a right triangle.

A ramp was built to add wheelchair access to a public building. The ramp rises 2 feet, as shown in the diagram below.



TEST-TAKING TIPS

Ensure that you're working with a right triangle before attempting to use the Pythagorean Theorem. It only applies to right triangles.

a 1. If the lower edge of the ramp is 10 feet from the base of the building along level ground, what is the approximate length, in feet, of the ramp?

A. 9.2
B. 9.6
C. 9.8
D. 10.2

b 2. The owners of the building are remodeling the front entrance. They would like to modify the ramp so that it begins 12 feet from the building. What will be the length of this new ramp?

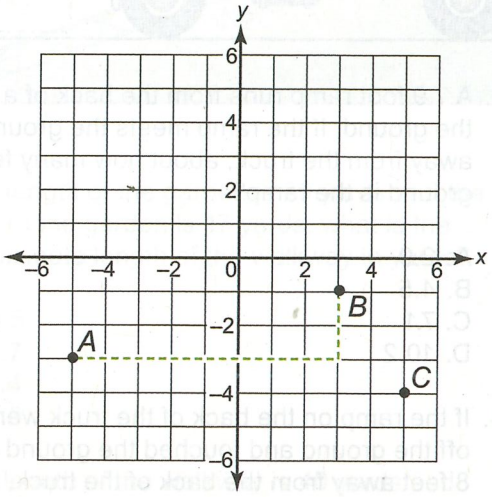
A. 11.8 ft
B. 12.2 ft
C. 12.4 ft
D. 12.5 ft

UNIT 4

3 Master the Skill

Spotlighted Item: FILL-IN-THE-BLANK

DIRECTIONS: Study the coordinate plane, read each question, and fill in your answer in the box below.



3. What is the distance between points *A* and *B*? Round your answer to the nearest hundredth.

4. What is the distance between points *A* and *C*? Round your answer to the nearest hundredth.

5. A new shape, Δ*JKL* is drawn on the given coordinates: *J* (−4, 4) *K* (−4, 0) *L* (2, 0). What is the distance between points *J* and *L*? Round your answer to the nearest hundredth.

6. What is the distance between points *K* and *L*?

7. What is the perimeter of Δ*JKL*? Round your answer to the nearest hundredth.

DIRECTIONS: Read the information and question, and then choose the **best** answer.

Ella incorrectly determined the length of one of the legs of a right triangle. Her work is shown below.

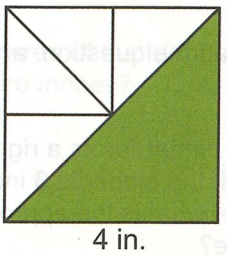
$$4^2 + a^2 = 10^2$$
$$16 + a^2 = 100$$
$$a^2 = 116$$
$$a \approx 10.8$$

8. Which answer choice best describes why Ella's answer is incorrect?

A. She incorrectly found the square root of 116.
B. She incorrectly squared 10.
C. She incorrectly squared 4.
D. She incorrectly added 16 when she should have subtracted.

DIRECTIONS: Read the figure and information below. Then read the question, and choose the **best** answer.

Dana designed the quilt square shown below.



9. What is the length of the diagonal of the outside square? Round your answer to the nearest hundred.

A. 5.66 in.
B. 4.00 in.
C. 2.83 in.
D. 2.01 in.

UNIT 4

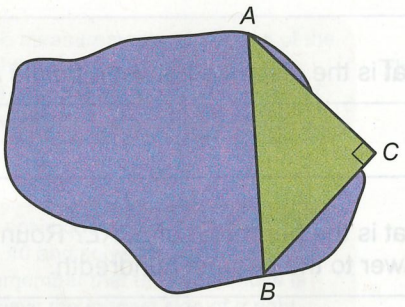
3 Master the Skill

DIRECTIONS: Read the question, and choose the **best** answer.

10. A computer monitor is listed as measuring 21 inches This is the distance across the diagonal of the screen. If the screen is 16 inches wide, what is the height of the screen to the nearest tenth of an inch?

    A. 4.5
    B. 9.1
    C. 13.6
    D. 27.2

DIRECTIONS: Study the figure below, read the question, and choose the **best** answer.



11. A surveyor wants to find the width of the pond. She placed stakes at points *A*, *B*, and *C*. She knows that ΔABC is a right triangle. If the distance between *A* and *C* is 75 feet and the distance between *B* and *C* is 63 feet, what is the approximate width of the pond between points *A* and *B*?

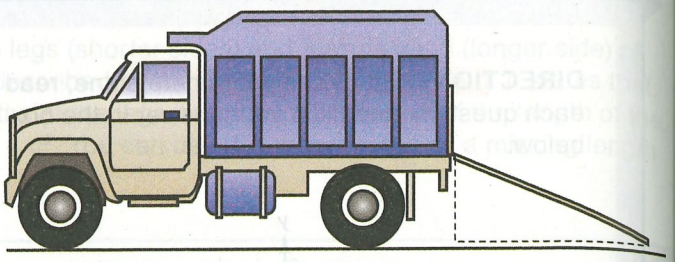
    A. 12.2 feet
    B. 40.7 feet
    C. 54.5 feet
    D. 97.9 feet

DIRECTIONS: Read the question, and choose the **best** answer.

12. Caleb's empty binder forms a right triangle from the side view. If the binder is 3 inches tall and 11 inches wide, what is the approximate length of the hypotenuse?

    A. 8 in.
    B. 9.2 in.
    C. 11.4 in.
    D. 14 in.

DIRECTIONS: Study the diagram, read each question, and choose the **best** answer.



13. A 7.9 foot ramp runs from the back of a truck to the ground. If the ramp meets the ground 6.5 feet away from the truck, about how many feet off the ground is the ramp?

    A. 2.0
    B. 4.5
    C. 7.1
    D. 10.2

14. If the ramp on the back of the truck were 5 feet off the ground and touched the ground at a point 8 feet away from the back of the truck, about how many feet long would the ramp be?

    A. 3.6
    B. 5.3
    C. 9.1
    D. 9.4

DIRECTIONS: Read the question, and choose the **best** answer.

15. A right triangle has sides of 55 inches and 40 inches. What is the length of the hypotenuse to the nearest foot?

    A. 5
    B. 6
    C. 7
    D. 8

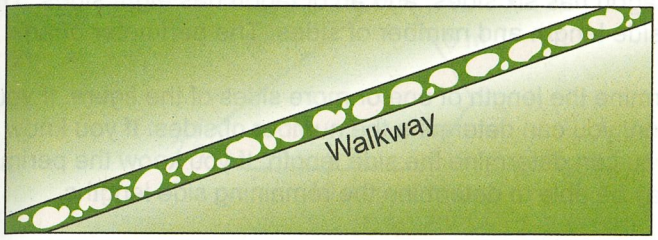
16. Laptop monitors are measured diagonally. Mona's laptop monitor measures 6 inches tall by 8.5 inches wide. What is her monitor's measurement?

    A. 10 inches
    B. 10.2 inches
    C. 10.4 inches
    D. 10.8 inches

UNIT 4

DIRECTIONS: Study the information and diagram, read each question, and choose the **best** answer.

Henry is building a walkway through a rectangular garden as shown in the diagram below.



17. If the length of the garden is 30 yards and the width of the garden is 17 yards, what is the approximate length of the walkway in yards?

    A. 34.5
    B. 24.7
    C. 21.4
    D. 13.3

18. If the length of the walkway is 40 yards and the width of the garden is 12 yards, what is the approximate length of the garden in yards?

    A. 28.4
    B. 38.2
    C. 41.8
    D. 52.3

DIRECTIONS: Read each question, and choose the **best** answer.

19. A 15-foot ladder is placed against the side of a building so that it reaches 12 feet up the side of the building. How far away from the building is its base?

    A. 8 feet
    B. 9 feet
    C. 10 feet
    D. 11 feet

20. The ladder is extended up to 18 feet. It now reaches 16 feet up the side of the building. About how far away from the building is its base now?

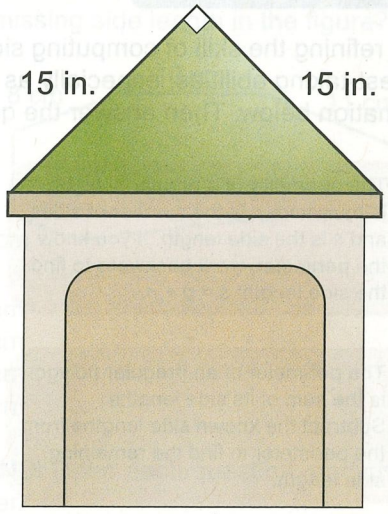
    A. 8.2 feet
    B. 9 feet
    C. 9.2 feet
    D. 10 feet

21. The 18-foot ladder is placed 6 feet from the side of the building. About how far up the building does it reach now?

    A. 13.4 feet
    B. 15.2 feet
    C. 16.97 feet
    D. 17.1 feet

DIRECTIONS: Study the figure and information below, read each question, and choose the **best** answer.

The front view of a dollhouse is shown below.



22. What is the approximate width of the dollhouse in inches?

    A. 17.5
    B. 18.6
    C. 19.2
    D. 21.2

23. What is the area of the triangular section of the roof in square inches?

    A. 112.5
    B. 159.0
    C. 225.0
    D. 318.0

24. If the height of the rectangular section is 18 inches, what is the perimeter of the front view?

    A. 69.2 inches
    B. 87.2 inches
    C. 90.4 inches
    D. 108.4 inches

UNIT 4