LESSON 3

# Squaring, Cubing, and Taking Roots

**MATH CONTENT TOPICS:** Q.2.a, Q.2.b, Q.2.c, Q.2.d, Q.2.e, A.1.e
**MATH PRACTICES:** MP.1.a, MP.1.b, MP.1.e, MP.2.c, MP.3.a, MP.4.a, MP.5.a, MP.5.b, MP.5.c

## 1 Learn the Skill

When a number or a variable is multiplied by itself, the result is called the **square** of that number or variable. Squaring the number 5, for example, is finding the product $5 \times 5 = 25$; this product is written as $5^2$, where the $^2$ indicates that the product is composed of two factors of 5.

When a number or variable is multiplied by itself an additional time, the result is called the **cube** of the number or variable. For example the cube of 5 is $5 \times 5 \times 5 = 125$; this product is written as $5^3$.

To find the **square root** of a number, find the number that, when squared, equals the given number. The **cube root** of a number is that number which, when cubed, equals the given number. Square and cube roots are indicated by root signs, such as $\sqrt{25}$ and $\sqrt[3]{125}$, respectively.

## 2 Practice the Skill

By practicing the skills of squaring, cubing, and taking the corresponding roots of quantities, you will improve your study and test-taking abilities, especially as they relate to the GED® Mathematical Reasoning Test. Study the information below. Then answer the question that follows.

**a** The square of a negative number is positive; if two numbers differ only in their sign, their squares are both positive and equal. The square root of a positive number can, as a result, have two values. Since there are no real numbers that, when multiplied by themselves, give a negative number, square roots of negative numbers are undefined when dealing with real numbers.

**b** The cube of a negative number is negative. As a result, the cube root of a negative number exists, is negative, and is equal in magnitude to the cube root of the absolute value of the number.

| | |
|---|---|
| $1^2 = 1 \times 1 = 1$ | $1^3 = 1 \times 1 \times 1 = 1$ |
| $2^2 = 2 \times 2 = 4$ | $2^3 = 2 \times 2 \times 2 = 8$ |
| $(-1)^2 = (-1) \times (-1) = 1$ | $(-1)^3 = (-1) \times (-1) \times (-1) = -1$ |
| $(-2)^2 = (-2) \times (-2) = 4$ | $(-2)^3 = (-2) \times (-2) \times (-2) = -8$ |
| $\sqrt{1} = 1, -1$ | $\sqrt[3]{1} = 1$ |
| $\sqrt{4} = 2, -2$ | $\sqrt[3]{8} = 2$ |
| $\sqrt{9} = 3, -3$ | $\sqrt[3]{27} = 3$ |
| $\sqrt{-9}$ = undefined | $\sqrt[3]{-27} = -3$ |

**TEST-TAKING TIPS**

*Taking the square root of a number is different from dividing a number by 2. When finding the square root of x, consider: what number times itself equals x? When dividing x by 2, think: what number plus itself equals x?*

1. The length of a square can be determined by finding the square root of its area. If a square has an area of 81 $m^2$, what is the length of the square?

   A. 8.0 m
   B. 8.5 m
   C. 9.0 m
   D. 9.5 m

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## 3 Apply the Skill

### Spotlighted Item: DRAG-AND-DROP

**DIRECTIONS:** Examine the information. Then read each question and use the drag-and-drop options to complete each answer.

2. Complete the following, assuming that $x < 0$.

$x^2$ [ ] $x^3$ [ ]

$\sqrt{x}$ [ ] $\sqrt[3]{x}$ [ ]

[> 0] [< 0] [= undefined]

3. Choose the solution (or solutions) of the equation $x^3 = 512$ from the list provided. The equation may have one solution, more than one solution, or no solution at all.

Solution(s): [ ]

[−16] [−8] [8] [16] [32]

**DIRECTIONS:** Read each question, and choose the **best** answer.

4. Carlos completed $x^3$ squats as part of his football workout. If $x = 5$, how many squats did he complete?

A. 10
B. 15
C. 25
D. 125

5. The length of a side of a cube can be determined by finding the cube root of its volume. If a cube has a volume of 64 $cm^3$, what is the length of a side?

A. 4.0 cm
B. 8.0 cm
C. 16.0 cm
D. 32.0 cm

6. To determine the length of yarn needed for a project, Josie must solve $\frac{\sqrt{x}}{4}$ for $x = 64$. What is the solution?

A. 2
B. 4
C. 8
D. 16

7. Mark multiplied a number by itself. He found a product of 30. What is the number, rounded to the nearest tenth?

A. 4.5
B. 5.4
C. 5.5
D. 15.0

8. A square has an area of 50 square feet. What is the perimeter of the square, rounded to the nearest foot?

A. 7 ft
B. 28 ft
C. 49 ft
D. 100 ft

9. A student is told that a moving car travels a distance $x$, where $x$, expressed in miles, is the solution of the equation $(8 - x)^2 = 64$. The student argues that the car can't be moving, since $x$ must equal zero. Which of the following describes the distance, $x$?

A. The student is correct; $x$ must equal zero.
B. $x$ is greater than zero, but less than 10
C. $x$ is between 10 and 20
D. $x$ is greater than 20

10. A gallon has a volume of 231 cubic inches. If a gallon of milk was sold in a perfectly cubical container, to the nearest tenth of an inch how high would the container be?

A. 6.0 inches
B. 6.1 inches
C. 6.2 inches
D. 6.3 inches

LESSON 4

# Exponents and Scientific Notation

**MATH CONTENT TOPICS:** Q.1.c, Q.2.a, Q.2.b, Q.2.c, Q.2.d, Q.2.e, Q.4.a, A.1.d, A.1.e, A.1.f
**MATH PRACTICES:** MP.1.a, MP.1.b, MP.1.e, MP.2.c, MP.3.a, MP.3.c, MP.4.a, MP.4.b, MP.5.c

## 1 Learn the Skill

**Exponents** are used when a number, called the base, is multiplied by itself many times. The exponent shows the number of times that the base appears in the product. When a quantity is given an exponent of *n*, it is said to be raised to the *n*th **power**; for example, $2^5$ is the same as 2 raised to the $5^{th}$ power. There are rules for adding, subtracting, multiplying, and dividing quantities with exponents.

**Scientific notation** uses exponents and powers of 10 to write very small and very large numbers in a compact form that simplifies calculations. Scientific notation requires that the decimal point be located just to the right of the first non-zero digit.

## 2 Practice the Skill

By practicing the skill of working with exponents and scientific notation, you will improve your study and test-taking abilities, especially as they relate to the GED® Mathematical Reasoning Test. Study the examples below. Then answer the question that follows.

**a** A number or quantity raised to the first power equals itself. A number or quantity (except zero) raised to the zero power equals one. When a number is raised to a negative power, write the reciprocal and change the negative exponent to a positive.

**b** Terms can be added and subtracted if they are alike, meaning they must have the same variable raised to the same exponent.

**c** To multiply terms with the same base, keep the base and add the exponents. Do the opposite for division. If the bases are not the same, simplify using the order of operations.

| | |
|---|---|
| $5^1 = 5$ $\quad 5^0 = 1$ **a** | $5^{-2} = \frac{1}{5^2} = \frac{1}{25}$ **a** |
| $2x^2 + 4x^2 + 1 = 6x^2 + 1$ **b** | $4x^2 - x^2 = 3x^2$ **b** |
| $(3^2)(3^3) = (3)^{2+3} = 3^5$ **c** | $\frac{6^5}{6} = 6^{5-1} = 6^4$ **c** |
| $4.2 \times 10^7 = 42{,}000{,}000$ | $5{,}800{,}000 = 5.8 \times 10^6$ |
| $3.7 \times 10^{-5} = 0.000037$ **d** | **d** $0.000052 = 5.2 \times 10^{-5}$ |

**d** To write a number shown in scientific notation as a number in expanded form, look at the power of 10. The exponent tells how many places to move the decimal point—right for positive, left for negative. To write a number in scientific notation, place the decimal point directly after the ones digit. Next, count the number of places you need to move. Then drop the zeros at the ends.

1. The distance between the sun and Mercury is about 58,000,000 km. What is this distance written in scientific notation?

   A. $5.8 \times 10^6$
   B. $5.8 \times 10^7$
   C. $58 \times 10^6$
   D. $58 \times 10^7$

**TEST-TAKING TECH**

The TI-30XS MultiView™ online calculator features a setting, using the SCI numeric notation mode, which allows calculations to be performed using scientific notation.

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## 3 Apply the Skill

**DIRECTIONS:** Read each question, and choose the **best** answer.

2. There are 25,400,000 nanometers in an inch. What is this number written in scientific notation?

   A. $2.54 \times 10^6$
   B. $2.54 \times 10^7$
   C. $2.54 \times 10^8$
   D. $2.54 \times 10^9$

3. The width of a rectangle is $2^6$, and the length is $2^5$. What is the area of the rectangle?

   A. $2^1$
   B. $2^{11}$
   C. $2^{30}$
   D. $4^{11}$

4. The width of a certain strand of human hair is about $1.5 \times 10^{-3}$ cm. What is the width of $2.0 \times 10^5$ of these hairs placed next to each other?

   A. $3.5 \times 10^8$ cm
   B. $3.0 \times 10^{-2}$ cm
   C. $3.0 \times 10^8$ cm
   D. $3.0 \times 10^2$ cm

5. Which has the same value as $5^1 + 4^0$?

   A. 9
   B. 8
   C. 6
   D. 5

6. Which of the following expressions is equivalent to $5(7^2 7^2) + 5(7^4 7^{-4}) - (7^8 7^{-4})$?

   A. $4(7^4) + 5$
   B. $6(7^8) + 1$
   C. $10(7^4) - (7^{-2})$
   D. $10(7^4) - (7^2)$

7. For what value of $x$ is the expression $(x^2 + 4)^2 (x^3 + 8)^{-3} (x^4 + 16)^4$ undefined?

   A. −8
   B. −4
   C. −2
   D. 2

**DIRECTIONS:** Read each question, and choose the **best** answer.

8. Which has the same value as $6(2^{-3}) + (5)(2^{-4}) + (4)(2^{-5})$?

   A. $\frac{15}{2}$
   B. $\frac{19}{8}$
   C. $\frac{19}{16}$
   D. −256

9. What is the answer to the equation $(3x^2 + 3x + 2) + 2(x^2 - 5x - 2)$?

   A. $(5x^2 - 2x)$
   B. $(5x^2 - 13x - 6)$
   C. $(5x^2 + 13x + 6)$
   D. $(5x^2 - 7x - 2)$

10. What is the answer to the equation $(3x^2 + 3x + 2) - 2(x^2 - 5x - 2)$?

    A. $(x^2 + 13x + 6)$
    B. $(x^2 - 7x - 2)$
    C. $(x^2 + 8x + 4)$
    D. $(x^2 - 2x)$

11. What is the answer to the equation $\frac{[(6x^2 + 4) - 2(2 - 3x^2)]}{4x}$?

    A. $\left(\frac{x}{2} + \frac{5}{2x}\right)$
    B. $3x$
    C. $\frac{2}{x}$
    D. $3x^2$

12. Max says that $x^2$ is always greater than $x^{-2}$. Which value of $x$ shows that Max is incorrect?

    A. $\frac{1}{3}$
    B. $\sqrt{3}$
    C. 3
    D. 30